

TECHNICAL NOTE

D-1223

A THEORETICAL INVESTIGATION OF THE
EFFECT OF CROSS-CONTROL DERIVATIVES ON THE STABILITY
CHARACTERISTICS OF AIRPLANES DESIGNED FOR FLIGHT
AT HIGH MACH NUMBERS

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SUMMARY

Linearized equations of motion were used to determine the lateral stability characteristics of a high-speed airplane with aerodynamic controls at a Mach number of 6 and an altitude of 125,000 feet. The period and damping of the oscillatory mode, the damping of the aperiodic modes, and the ratio of the roll angle to the sideslip angle were analyzed to determine the effects of the cross-control derivatives.

Results show that an increase in the roll-damper gain causes a relatively large increase in the damping with a moderate increase in the frequency of the Dutch roll mode as a result of the cross-control effectiveness. This could cause critical ratios of the roll angle to the sideslip angle if a negative effective dihedral exists. In addition, when damper gains are relatively large, the cross-control derivatives could overcome divergence that results from a negative effective dihedral and a small directional derivative.

INTRODUCTION

The results of some recent investigations of the lateral stability of high-speed, high-altitude airplanes show the necessity of auxiliary dampers to make them acceptable for flight.

When dampers are added to improve the lateral stability of high-speed airplanes, the cross-control effectiveness may have a destabilizing effect on the aircraft. Such occurrences have been noted in the Dutch roll mode when roll dampers are added and in the spiral mode when yaw dampers are added. References 1 and 2 present results which show these conditions.

An investigation was conducted to determine the effects of the cross-control derivatives on the lateral stability of high-speed airplanes for different roll-damper gains and combinations of the directional-stability parameter and the effective-dihedral parameter. The results are presented herein as plots of the lateral characteristics against the cross-control effectiveness, with satisfactory and unsatisfactory boundaries indicated.

SYMBOLS

b	span, ft
$C_{l/2}$	cycles to damp to half-amplitude
C_l	rolling-moment coefficient, $\frac{\text{Rolling moment}}{qSb}$
C_n	yawing-moment coefficient, $\frac{\text{Yawing moment}}{qSb}$
C_Y	side-force coefficient, $\frac{\text{Side force}}{qS}$
g	acceleration due to gravity, 32.2 ft/sec ²
h	altitude, ft
I_X	moment of inertia about the X principal body axis, slug-ft ²
I_Z	moment of inertia about the Z principal body axis, slug-ft ²
k_1	roll-damper gain, $\delta_a/\dot{\phi}$ (variable)
k_2	yaw-damper gain, $\delta_r/\dot{\psi}$ (constant)
M	Mach number
m	mass, slugs
p, r	angular velocity of airplane about the X and Z body axes
q	dynamic pressure $\frac{1}{2}\rho V^2$, lb/sq ft

S	wing area, sq ft
t	time, sec
$t_{1/2}$	time to damp to half-amplitude, sec
t_2	time to double amplitude, sec
V	velocity, ft/sec
α	angle of attack
β	angle of sideslip
$\dot{\beta}$	sideslip angular velocity, $\frac{d\beta}{dt}$
δ_a	aileron deflection
δ_r	rudder deflection
ρ	air density, slugs/cu ft
ϕ	angle of roll
$\dot{\phi}$	rolling velocity, $\frac{d\phi}{dt}$
$\ddot{\phi}$	rolling acceleration, $\frac{d^2\phi}{dt^2}$
ψ	angle of yaw
$\dot{\psi}$	yawing velocity, $\frac{d\psi}{dt}$
$\ddot{\psi}$	yawing acceleration, $\frac{d^2\psi}{dt^2}$
$C_{l_\beta} = \frac{\partial C_l}{\partial \beta}$	
$C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}$	

$$l_4$$

$$c_{l_r} = \frac{\partial c_l}{\partial \frac{rb}{2V}}$$

$$c_{l_{\delta a}} = \frac{\partial c_l}{\partial \delta_a}$$

$$c_{l_{\delta r}} = \frac{\partial c_l}{\partial \delta_r}$$

$$c_{l_{\dot{\phi}}} = \frac{b}{2V} \; c_{l_p}$$

$$c_{l_{\dot{\psi}}} = \frac{b}{2V} \; c_{l_r}$$

$$c_{n_{\beta}} = \frac{\partial c_n}{\partial \beta}$$

$$c_{n_p} = \frac{\partial c_n}{\partial \frac{pb}{2V}}$$

$$c_{n_r} = \frac{\partial c_n}{\partial \frac{rb}{2V}}$$

$$c_{n_{\delta a}} = \frac{\partial c_n}{\partial \delta_a}$$

$$c_{n_{\delta r}} = \frac{\partial c_n}{\partial \delta_r}$$

$$c_{Y_{\beta}} = \frac{\partial c_Y}{\partial \beta}$$

$$c_{n_{\dot{\phi}}} = \frac{b}{2V} \; c_{n_p}$$

$$c_{n_{\dot{\psi}}} = \frac{b}{2V} \; c_{n_r}$$

Subscripts:

dyn dynamic

eff effective

Unless otherwise noted, all angles are measured in radians and all angular velocities are in radians per second.

RESULTS AND DISCUSSION

A typical high-speed research airplane with aerodynamic controls was used to investigate the effects of the cross-control derivatives on the lateral stability. The flight condition was a trimmed straight and level flight at a Mach number of 6 and an altitude of 125,000 feet. At this speed and altitude the trim angle of attack was 20° . The airplane was represented by linearized equations of lateral motion referred to the principal body axes. The following equations were used to calculate the lateral stability characteristics:

$$\ddot{\phi} - \frac{qSb}{I_X} k_1 C_{l_{\delta a}} \dot{\phi} - \frac{qSb}{I_X} k_2 C_{l_{\delta r}} \dot{\psi} - \frac{qSb}{I_X} C_{l_{\beta}} \beta = 0 \quad (1)$$

$$- \frac{qSb}{I_Z} k_1 C_{n_{\delta a}} \dot{\phi} + \ddot{\psi} - \frac{qSb}{I_Z} k_2 C_{n_{\delta r}} \dot{\psi} - \frac{qSb}{I_Z} C_{n_{\beta}} \beta = 0 \quad (2)$$

$$-\alpha \dot{\phi} - \frac{g}{V} \phi + \dot{\psi} + \dot{\beta} - \frac{qS}{mV} C_{Y_{\beta}} \beta = 0 \quad (3)$$

The airplane was assumed to have negligible natural damping. The auxiliary dampers which account for all damping were represented by introducing effective damping derivatives which are defined by the following equations:

$$(C_{l_p})_{\text{eff}} = \frac{2V}{b} k_1 C_{l_{\delta a}} \quad (4)$$

$$(C_{l_r})_{\text{eff}} = \frac{2V}{b} k_2 C_{l_{\delta r}} \quad (5)$$

$$(C_{np})_{\text{eff}} = \frac{2V}{b} k_1 C_{n\delta a} \quad (6)$$

$$(C_{nr})_{\text{eff}} = \frac{2V}{b} k_2 C_{n\delta r} \quad (7)$$

Calculations were made to determine the period and damping of the Dutch roll mode, the damping of the roll and spiral modes, and the ratio of roll angle ϕ to sideslip angle β . These lateral stability characteristics were determined for a range of cross-control derivatives $C_{n\delta a}$ and $C_{l\delta r}$ with the combinations of $C_{n\beta}$, $C_{l\beta}$, and roll-damper gain k_1 presented in table I. A yaw-damper gain k_2 which provided good Dutch roll characteristics for $C_{n\delta a} = C_{l\delta r} = 0$ was selected; this value of yaw-damper gain ($k_2 = 0.366$) was maintained throughout the investigation. The airplane characteristics and flight conditions are presented in table II.

In order to define the characteristics of a satisfactory airplane, references 3 and 4 were used to set up the criteria for the Dutch roll oscillation and the ratio ϕ/β . It was determined that the Dutch roll characteristics would be satisfactory for $1/C_{l1/2} \geq 0.7$ and $\phi/\beta < 4$, tolerable for $0.7 > 1/C_{l1/2} > 0.25$, and unsatisfactory for $1/C_{l1/2} < 0.25$. The damping-in-roll and spiral modes were arbitrarily selected as being satisfactory for $1/t_{1/2} \geq 1$ and $1/t_2 < 0.1$, respectively. These criteria were used to establish stability boundaries (for example, see fig. 1), with the region between the Dutch roll curve for $1/C_{l1/2} = 0.7$ and the damping-in-roll curve for $1/t_{1/2} = 1$ representing a domain of satisfactory lateral stability. To show spiral divergence, the curve $1/t_2 = 0$ is included.

Effects of Varying the Roll-Damper Gain k_1

Figure 1(a) shows the satisfactory range of the lateral stability characteristics for a roll-damper gain of $k_1 = 0.05$; this range is limited mainly by $C_{l\delta r}/C_{n\delta r}$ with little restriction from $C_{n\delta a}/C_{l\delta a}$. For a value of $C_{n\delta a}/C_{l\delta a} = 0.8$, the Dutch roll oscillation is satisfactory for $C_{l\delta r}/C_{n\delta r} \leq 0.1$, tolerable for $0.1 < C_{l\delta r}/C_{n\delta r} \leq 0.25$, and

unsatisfactory for $C_{l\delta r}/C_{n\delta r} > 0.25$. The damping in roll is satisfactory for $C_{l\delta r}/C_{n\delta r} \geq 0.02$ and unsatisfactory for $C_{l\delta r}/C_{n\delta r} < 0.02$. The spiral mode becomes divergent for $C_{l\delta r}/C_{n\delta r} \leq -0.084$ and reaches an unsatisfactory condition for $C_{l\delta r}/C_{n\delta r} < -0.36$.

Increasing the roll-damper gain to $k_1 = 0.2$ (fig. 1(b)) increases the satisfactory range of $C_{l\delta r}/C_{n\delta r}$ in both a positive and a negative direction. With an increase in the damping of the roll mode, the damping of the Dutch roll and spiral modes also increases as a result of the cross-control effectiveness. As $C_{n\delta a}/C_{l\delta a}$ increases, the range of $C_{l\delta r}/C_{n\delta r}$ decreases for satisfactory damping of the Dutch roll mode.

To show the effects of $C_{l\delta r}/C_{n\delta r}$ on the lateral motion, and the change with an increase in the roll-damper gain, four sample points were selected as shown in figures 1(a) and 1(b), and time histories were made of the sideslip and yawing velocity responses. (See figs. 2 and 3.)

Figure 1(a) shows that a value of $C_{l\delta r}/C_{n\delta r} = 0.3$ with a roll-damper gain k_1 of 0.05 is in an unsatisfactory range for Dutch roll stability. The responses for this ratio (figs. 2(a) to 2(d)), show very little damping in the oscillation of the sideslip and yawing velocity.

When the rudder control derivative $C_{l\delta r}$ increases so that $C_{l\delta r}/C_{n\delta r} = 0.06$ (fig. 1(a)), the damping of the Dutch roll mode increases and a satisfactory condition for lateral stability exists. The responses (figs. 2(a) to 2(d)) show a satisfactorily damped oscillation for sideslip and yawing velocity.

A further increase in the rudder control derivative $C_{l\delta r}$, so that $C_{l\delta r}/C_{n\delta r} = -0.4$ (fig. 1(a)), results in an unsatisfactory condition in the roll and spiral modes. The responses (figs. 2(a) to 2(d)) become divergent for sideslip and yawing velocity.

With an increase in the roll-damper gain to $k_1 = 0.2$, a ratio of $C_{l\delta r}/C_{n\delta r} = 0.6$ (fig. 1(b)) is a questionable value for satisfactory Dutch roll stability. The responses (figs. 3(a) to 3(d)) show a damped oscillation for the sideslip and yawing velocity but the damping is not great enough to be considered satisfactory.

For a ratio of $C_{l\delta r}/C_{n\delta r} = 0.3$ an increase in the roll-damper gain to $k_1 = 0.2$ results in satisfactory lateral stability (fig. 1(b)). The responses (figs. 3(a) to 3(d)) are satisfactorily damped oscillations in sideslip and yawing velocity.

A ratio of $C_{l\delta r}/C_{n\delta r} = -0.4$ (fig. 1(b)) is also a satisfactory value for lateral stability when the roll-damper gain is increased. The responses (figs. 3(a) to 3(d)) are well-damped oscillations reaching a steady-state value in 4 seconds.

Effects of $C_{l\beta}$

Figure 4(a) shows that a negative effective dihedral causes very little change in the $C_{l\delta r}/C_{n\delta r}$ range for Dutch roll stability when $k_1 = 0.05$. The comparatively large value of $C_{n\beta}$ reduces the adverse effects of $C_{l\beta} > 0$, as can be seen from equation (A1) in the appendix. Equation (A1) is taken from the expression for the undamped natural frequency of the Dutch roll mode.

A negative effective dihedral reduces the $C_{l\delta r}/C_{n\delta r}$ range for Dutch roll stability when the roll-damper gain k_1 is 0.2 (fig. 4(b)). The comparatively large increase in damping with only a moderate increase in the frequency of the Dutch roll mode causes critical ϕ/β ratios, as mentioned in the appendix; this limits the $C_{l\delta r}/C_{n\delta r}$ range for satisfactory lateral stability.

Effects of $C_{n\beta}$

Decreasing the directional derivative with a positive effective dihedral (fig. 5(a)) increases the $C_{l\delta r}/C_{n\delta r}$ range for Dutch roll stability when $k_1 = 0.05$. The decrease in $C_{n\beta}$ results in more effective damping of the Dutch roll mode and an increase in the range of satisfactory lateral stability.

A decreased directional derivative with a roll-damper gain k_1 of 0.2 and a positive effective dihedral (fig. 5(b)) produces a large increase in the $C_{l\delta r}/C_{n\delta r}$ range for the Dutch roll stability. The satisfactory range of the lateral stability increases accordingly. As

$C_{n\delta a}/C_{l\delta a}$ increases, the product $C_{n\delta a}C_{l\delta r}$ becomes positive, causing a decrease in the damping and a small increase in the frequency of the Dutch roll mode. (See appendix.) The result is a decrease in the $C_{l\delta r}/C_{n\delta r}$ range for lateral stability.

For a decreased directional derivative with a negative effective dihedral (see fig. 6(a)), the $C_{l\delta r}/C_{n\delta r}$ range collapses for lateral stability. The dynamic directional derivative $(C_{n\beta})_{dyn}$ becomes negative and causes divergence in the Dutch roll mode. (See appendix.) The effective damping derivatives do not overcome the negative $(C_{n\beta})_{dyn}$ because of the small roll-damper gain. The stability boundaries of the Dutch roll and damping-in-roll modes become almost coincident. The reduced frequency causes large values of ϕ/β for positive values of $C_{l\delta r}/C_{n\delta r}$.

For a decreased directional derivative and a negative effective dihedral, increasing the roll-damper gain to $k_1 = 0.2$ overcomes the negative $(C_{n\beta})_{dyn}$ (fig. 6(b)). For positive values of $C_{n\delta a}/C_{l\delta a}$ a small lateral range of $C_{l\delta r}/C_{n\delta r}$ exists for satisfactory stability because of a negative $C_{n\delta a}C_{l\delta r}$, as shown in the appendix.

As $C_{n\delta a}/C_{l\delta a}$ decreases, the product $C_{n\delta a}C_{l\delta r}$ becomes positive and the primary factors of the Dutch roll spring constant become negative, causing divergence. The increase in the damping and frequency of the Dutch roll mode with the increase in the roll-damper gain reduces the value of $C_{l\delta r}/C_{n\delta r}$ for critical values of ϕ/β .

The foregoing analysis considers automatic damping to provide satisfactory levels of Dutch roll damping. For configurations with low Dutch roll damping, consideration should be given to certain roll-control problems experienced by the pilot (refs. 5 and 6) which impose restrictions on the configuration through the parameter $C_{n\delta a}C_{l\beta}/C_{l\delta a}C_{n\beta}$.

CONCLUDING REMARKS

The lateral stability characteristics of a high-speed airplane flying at a Mach number of 6, an altitude of 125,000 feet, and a trim angle of attack of 20° were determined as a function of roll- and yaw-damper gains and control-surface effectiveness. The period and damping

of the oscillatory mode, the damping of the aperiodic modes, and the ratio of roll angle to sideslip angle were analyzed to determine the effects of the cross-control derivatives on the lateral stability.

Results show that an increase in the roll-damper gain causes a relatively large increase in the damping with a moderate increase in the frequency of the Dutch roll mode as a result of the cross-control effectiveness. This could cause critical ratios of roll angle to sideslip angle if a negative effective dihedral exists. In addition, when damper gains are relatively large the cross-control derivatives could overcome divergence that results from a negative effective dihedral and a small directional derivative. These results indicate that each high-speed airplane configuration should be investigated to avoid divergence and adverse effects of dampers.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Air Force Base, Va., January 5, 1962.

APPENDIX

THE EFFECTS OF $C_{l\beta}$ AND THE CROSS-CONTROL DERIVATIVES
ON THE DUTCH ROLL MODE

As indicated in reference 2, the effective dihedral and large inertia ratios can have predominant effects on the Dutch roll mode. The contribution of a negative effective dihedral can be best seen in the primary factors of the undamped natural frequency of the Dutch roll mode. Negative values of the parameter

$$(C_{n\beta})_{\text{dyn}} = C_{n\beta} - \frac{I_Z}{I_X} \alpha C_{l\beta} \quad (\text{A1})$$

may lead to divergence.

A negative effective dihedral will decrease the frequency of the Dutch roll oscillation and may overcome directional stability if $C_{n\beta}$ is comparatively small. A sufficiently large $C_{n\beta}$ will decrease the effects of $C_{l\beta} > 0$.

The effects of the cross-control derivative on the Dutch roll mode can be a contributing factor, depending on the effective dihedral and damper gains, as can be seen by including the rotary derivatives of the Dutch roll spring constant with $(C_{n\beta})_{\text{dyn}}$:

$$C_{n\beta} - \frac{I_Z}{I_X} \alpha C_{l\beta} - \frac{qSb}{I_X} (C_{n\dot{\phi}} C_{l\dot{\psi}} - C_{l\dot{\phi}} C_{n\dot{\psi}}) \quad (\text{A2})$$

When the effective damping derivatives defined by equations (4) to (7) are introduced, equation (A2) becomes

$$C_{n\beta} - \frac{I_Z}{I_X} \alpha C_{l\beta} - \frac{qSb}{I_X} k_1 k_3 (C_{n\delta a} C_{l\delta r} - C_{l\delta a} C_{n\delta r}) \quad (\text{A3})$$

The preceding expression can be seen as a part of the coefficient of the square term in the quartic equation obtained from equations (1), (2), and (3). As the expression (A3) becomes zero, the Dutch roll

oscillatory mode becomes neutrally stable. A negative value of the cross-control derivative $C_{n\delta_a} C_{l\delta_r}$ would increase the damping and decrease the frequency of the Dutch roll mode, and a positive value would decrease the damping and increase the frequency. A negative product of the cross-control derivatives could overcome divergence that results from a negative effective dihedral and a small $C_{n\beta}$.

An increase in the roll-damper gain k_1 causes a large increase in the damping with only a moderate increase in the frequency of the Dutch roll mode. An increase in the roll-damper gain will help overcome divergence provided the cross-control derivatives have the correct sign. A large increase in the damping without sufficient increase in the frequency may lead to critical ratios of roll angle to sideslip angle.

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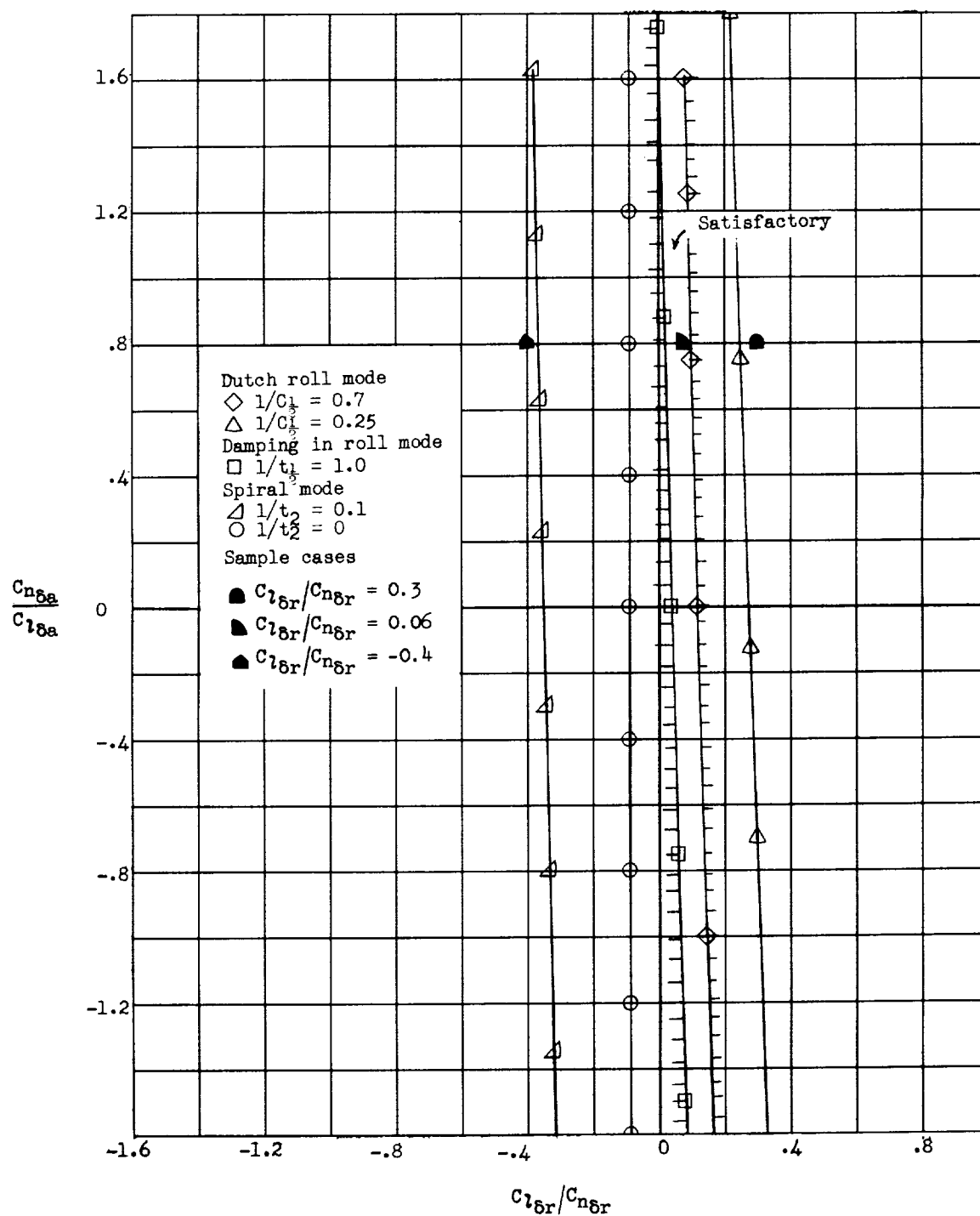
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TABLE I
STABILITY DERIVATIVES AND DAMPER-GAIN COMBINATIONS

$C_{n\beta}$	$C_{l\beta}$	k_1	k_2
0.31	-0.027	0.05	0.366
.31	-.027	.2	.366
.31	.027	.05	.366
.31	.027	.2	.366
.1	-.027	.05	.366
.1	-.027	.2	.366
.1	.027	.05	.366
.1	.027	.2	.366

TABLE II
AIRPLANE CHARACTERISTICS AND FLIGHT CONDITIONS

m, slugs	390.4
I_X , slug-ft ²	5,021
I_Z , slug-ft ²	67,199
b, ft	22.36
S, sq ft	200
h, ft	125,000
V, ft/sec	6,000
q, lb/sq ft	200
g, ft/sec ²	32.2
α , deg	20
$C_{l_{\delta a}}$, per radian	-0.075
$C_{n_{\delta r}}$, per radian	-0.108
$C_{Y_{\beta}}$, per radian	-1
$C_{n_{\beta}}$, per radian	0.31, 0.1
$C_{l_{\beta}}$, per radian	0.027, -0.027



(a) Roll-damper gain of $k_1 = 0.05$.

Figure 1.- Effect of cross-control derivatives on the lateral characteristics of a high-speed airplane with positive effective dihedral and automatic dampers. $k_2 = 0.366$; $M = 6$; $h = 125,000$ feet; $C_{l\beta} = -0.027$; $C_{n\beta} = 0.31$.

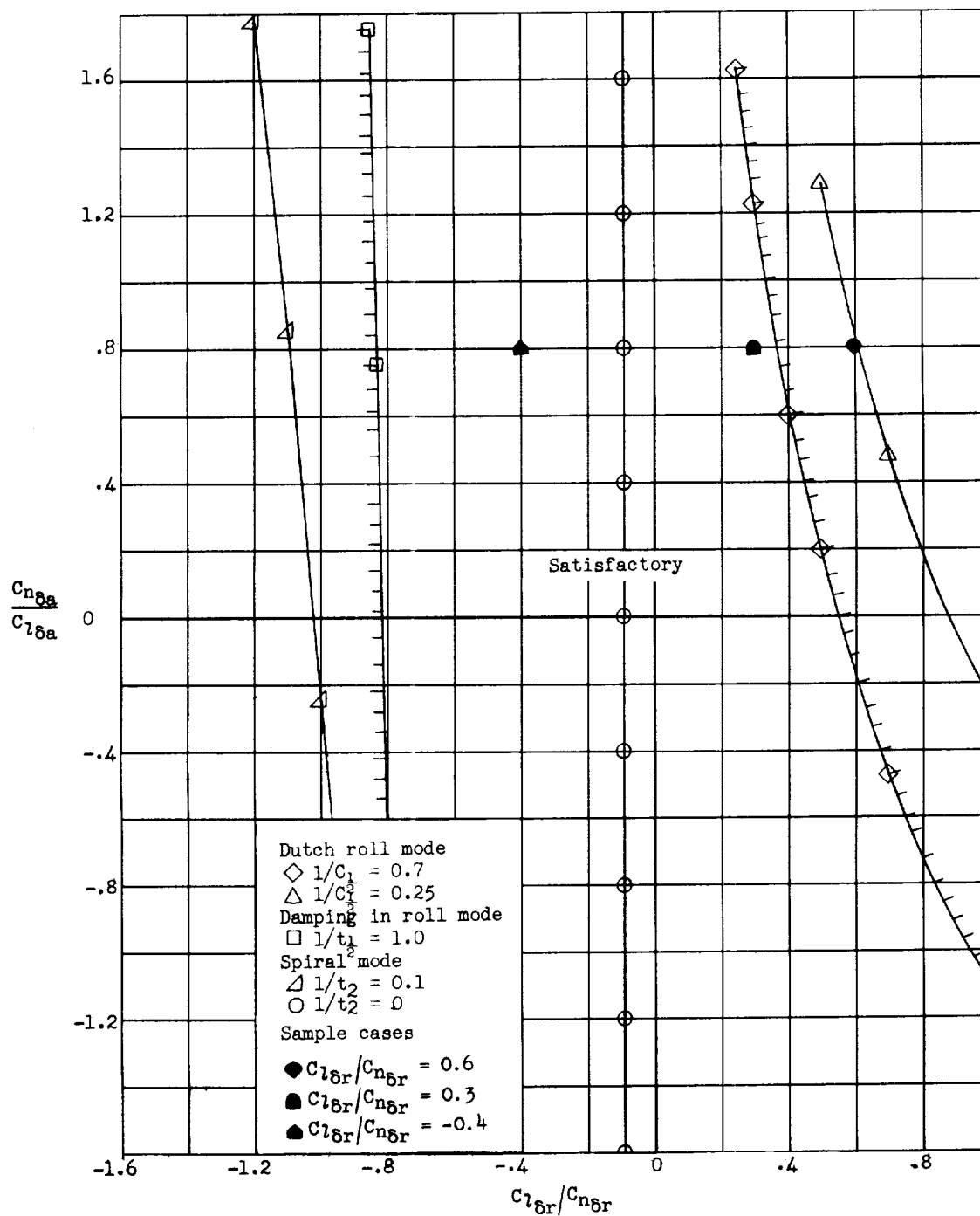
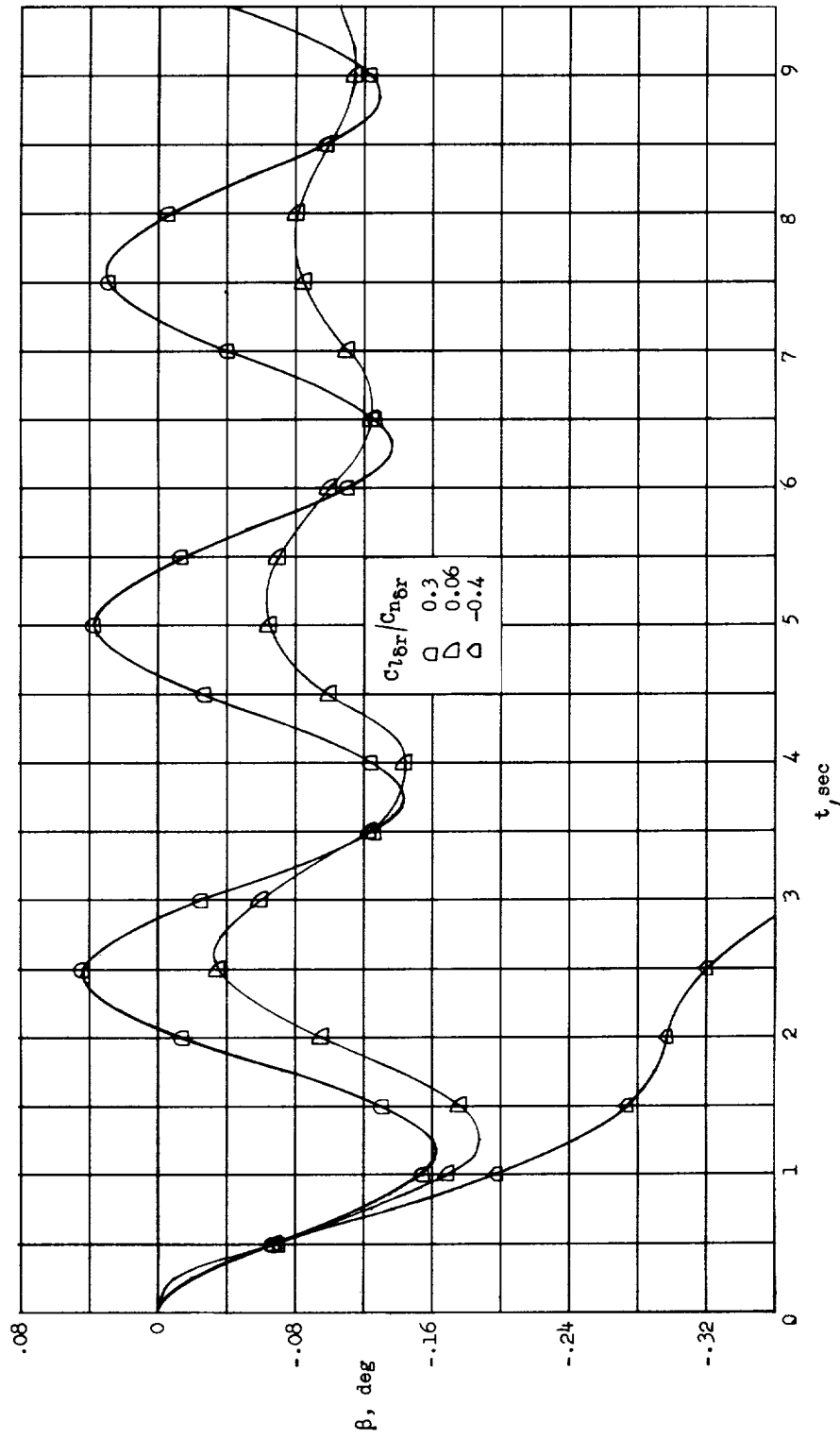
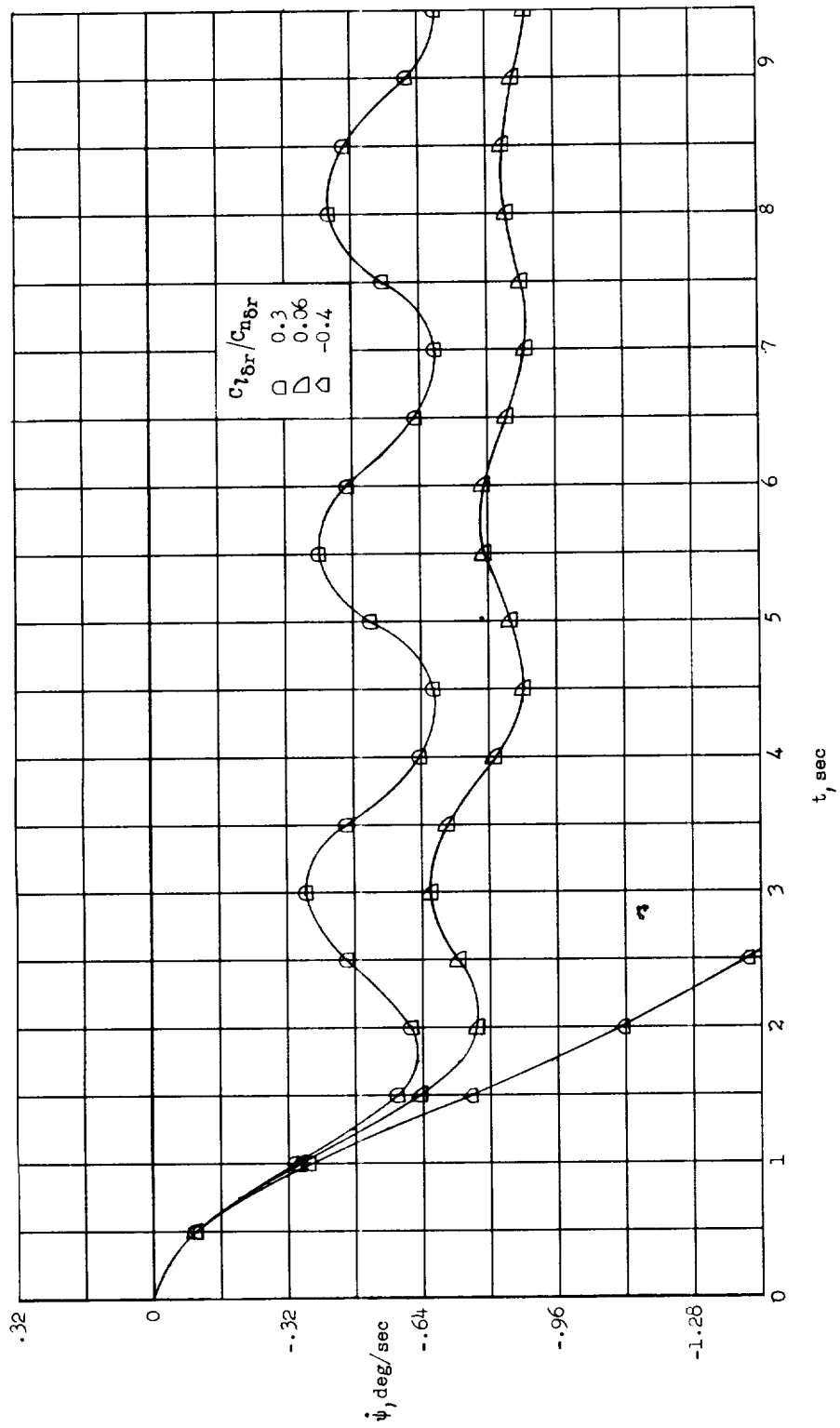
(b) Roll-damper gain of $k_1 = 0.2$.

Figure 1.- Concluded.



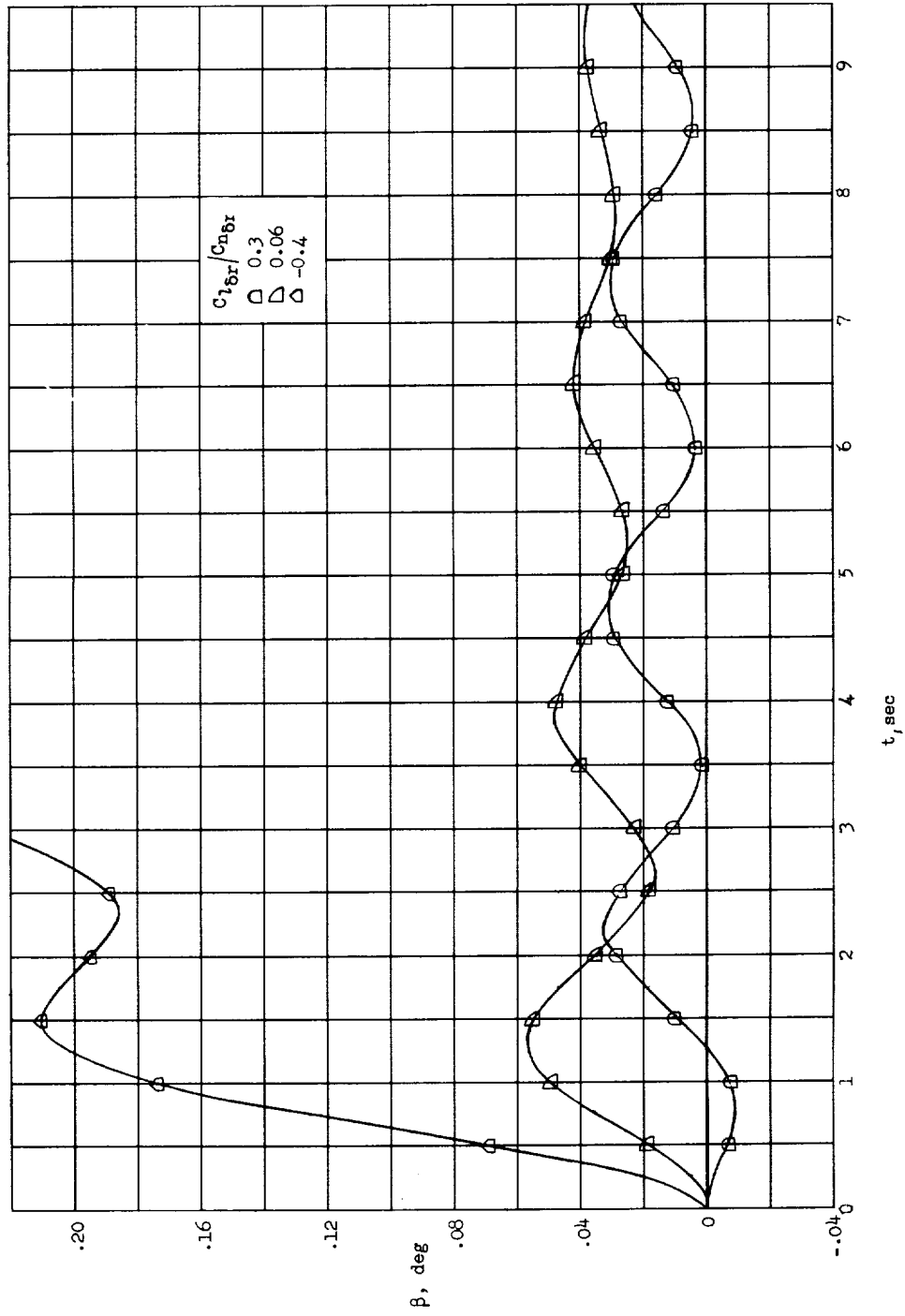
(a) Sideslip response to a 10° aileron step function.

Figure 2.- Response of a high-speed aircraft with positive effective dihedral and damper gains of $k_1 = 0.05$ and $k_2 = 0.366$; $C_{n\delta_a}/C_{l\delta_a} = 0.8$; $M = 6$; $h = 125,000$ feet; $C_{l\beta} = -0.027$; $C_{n\beta} = 0.31$.



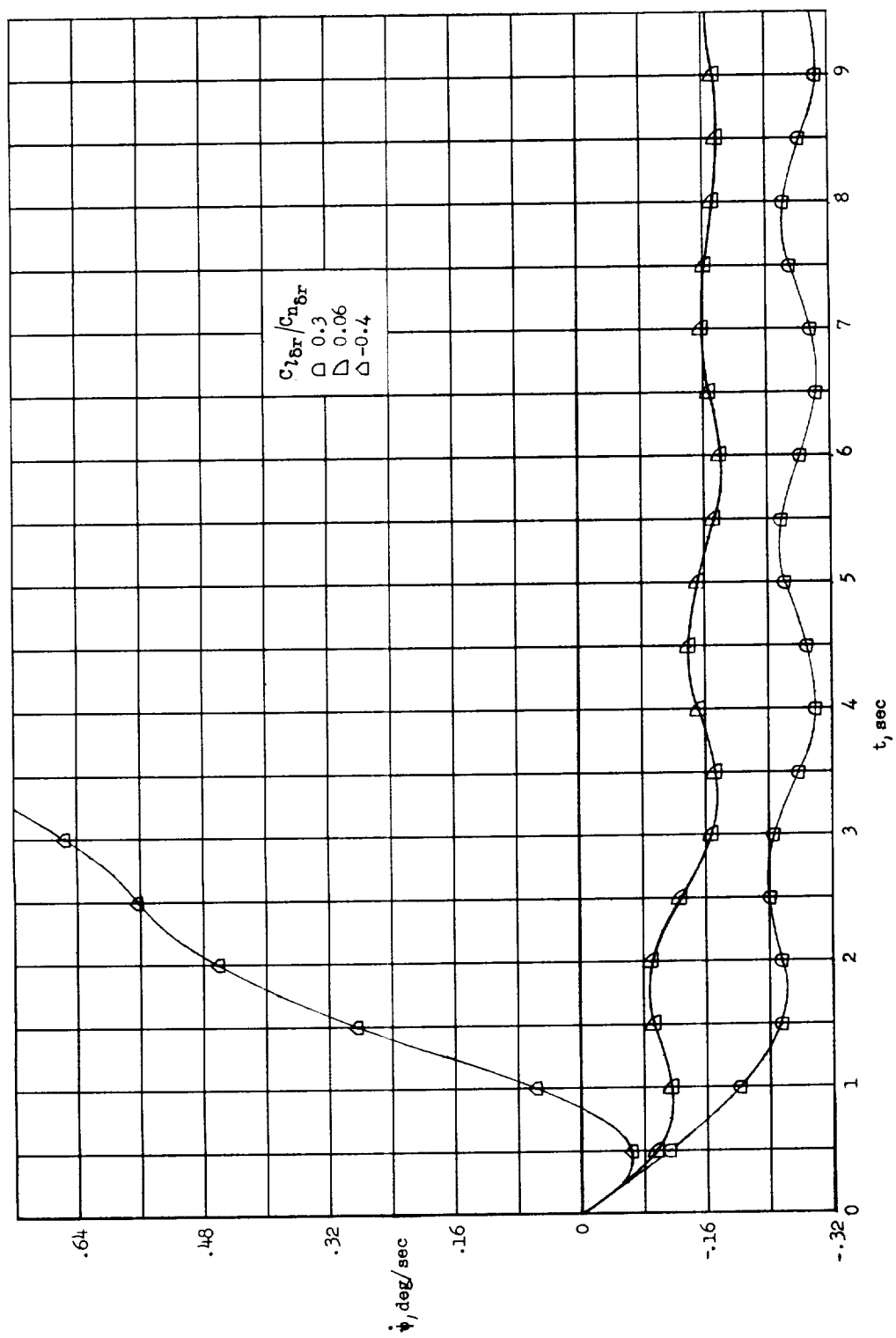
(b) Yawing-velocity response to a 10° aileron step function.

Figure 2.- Continued.



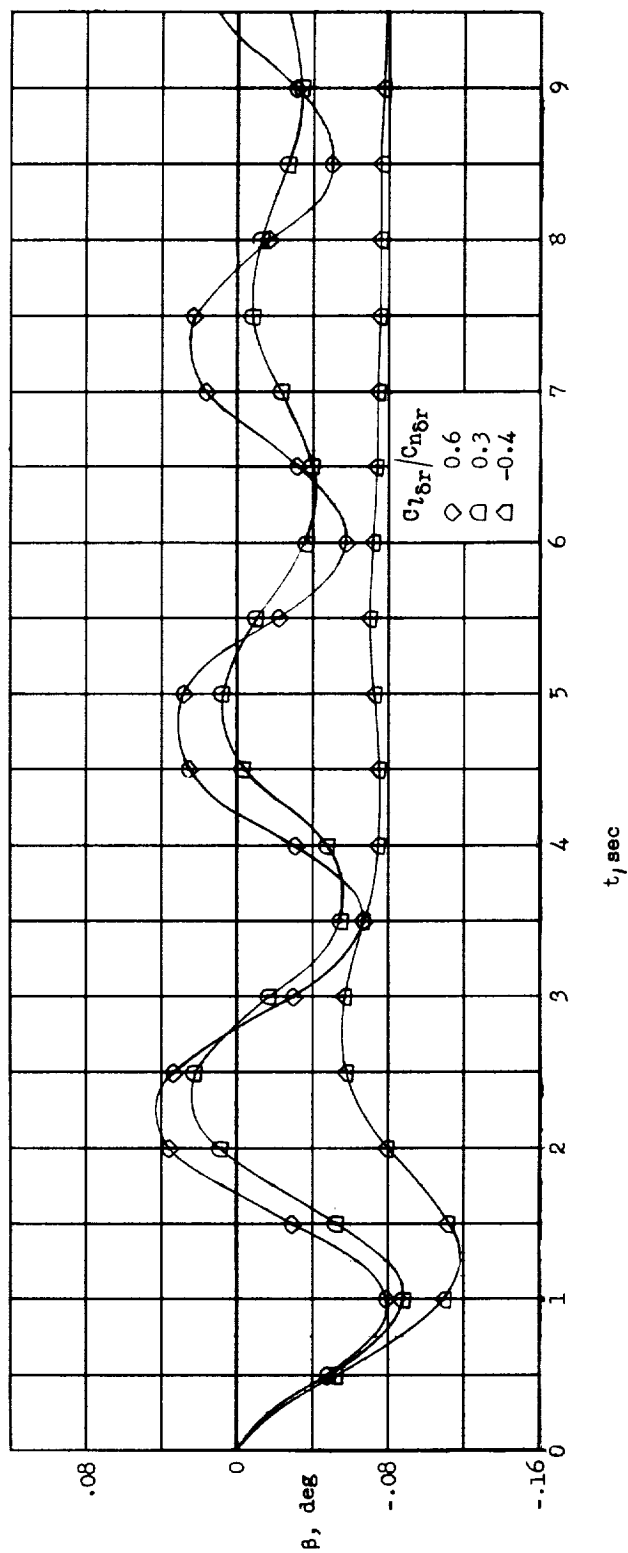
(c) Sideslip response to a 10° rudder step function.

Figure 2.- Continued.



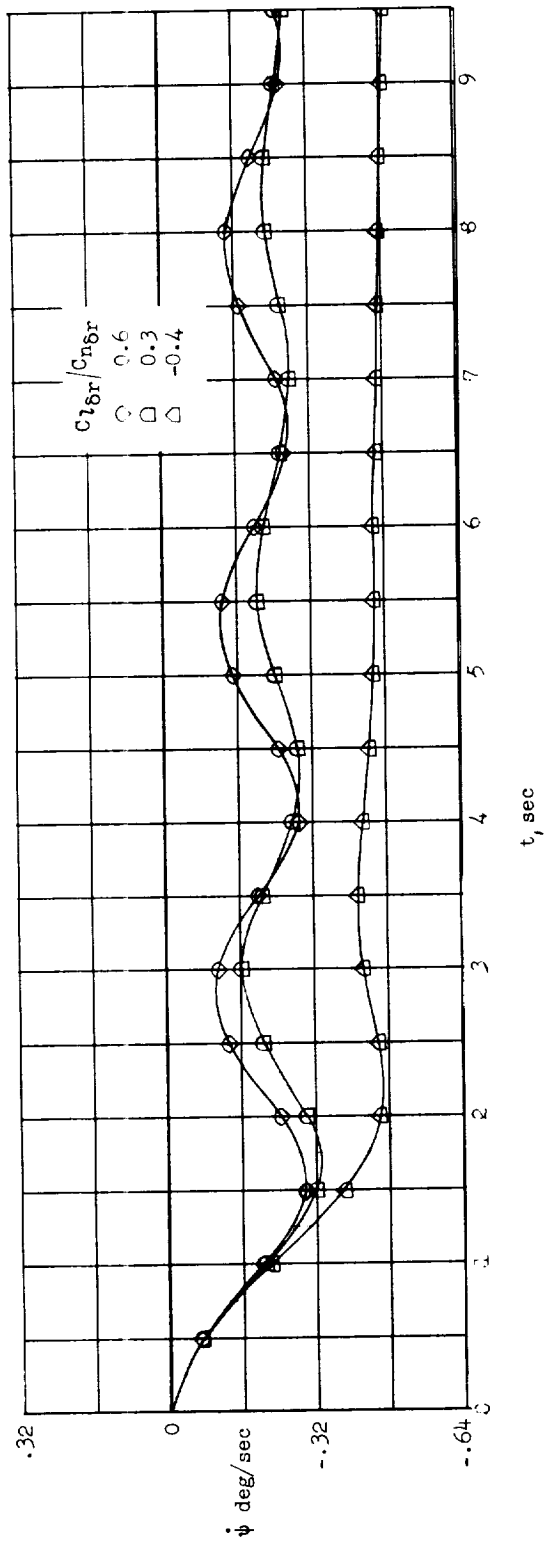
(d) Yawing-velocity response to a 10° rudder step function.

Figure 2.- Concluded.



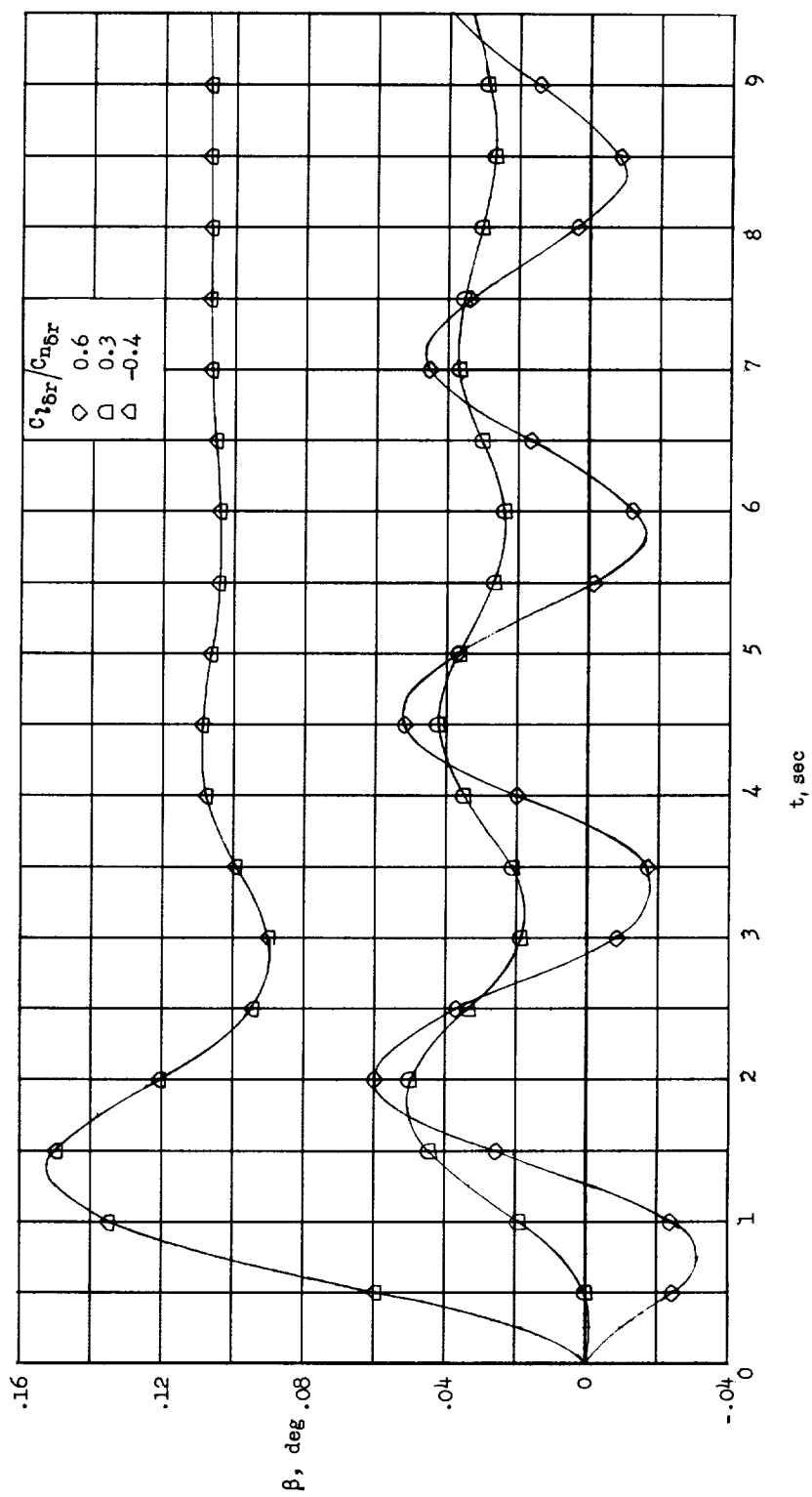
(a) Sideslip response to a 10° aileron step function.

Figure 3.- Response of a high-speed aircraft with positive effective dihedral and damper gains of $k_1 = 0.2$ and $k_2 = 0.366$; $C_{n\delta_a}/C_{l\delta_a} = 0.8$; $M = 6$; $h = 125,000$ feet; $C_{l\beta} = -0.027$; $C_{n\beta} = 0.31$.



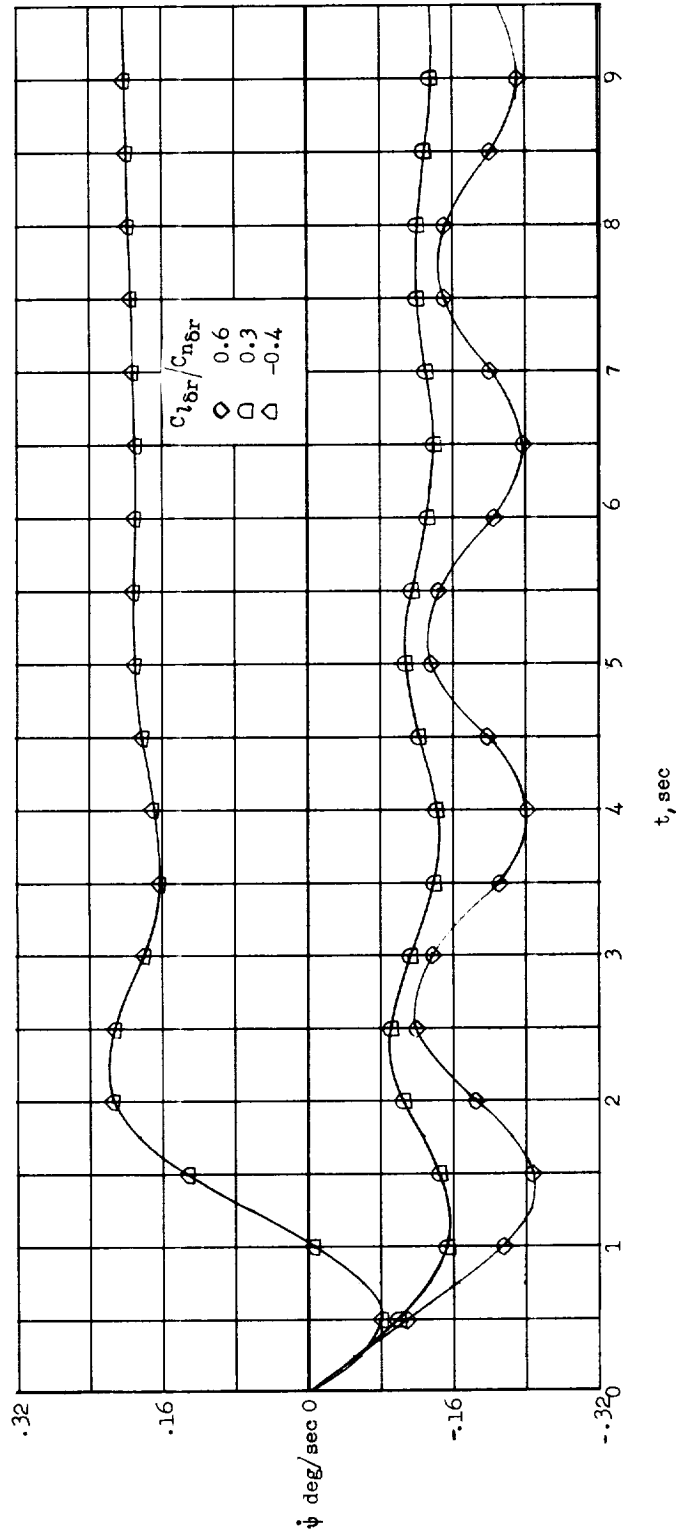
(b) Yawing-velocity response to a 10° aileron step function.

Figure 3.- Continued.



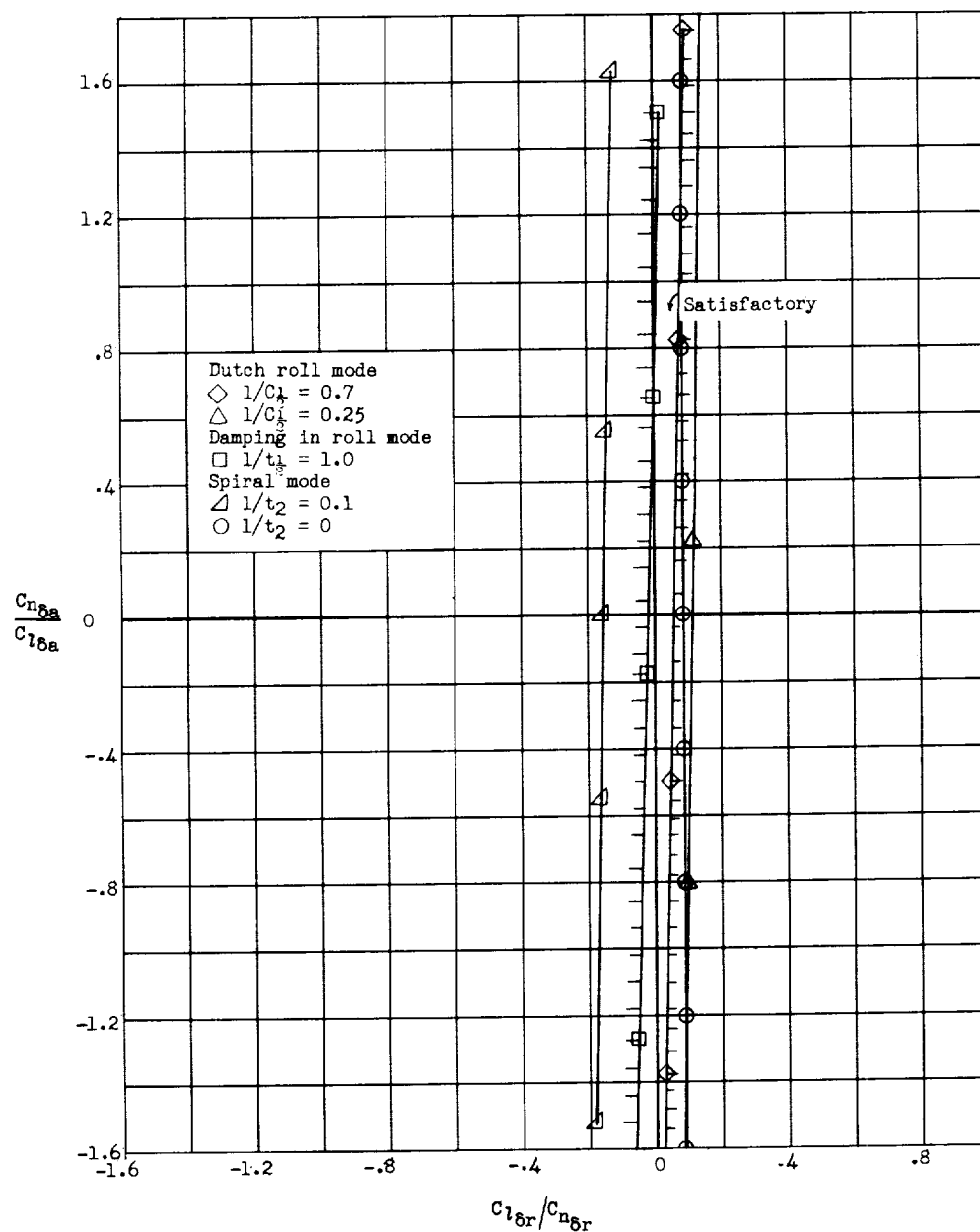
(c) Sideslip response to a 10° rudder step function.

Figure 3.- Continued.



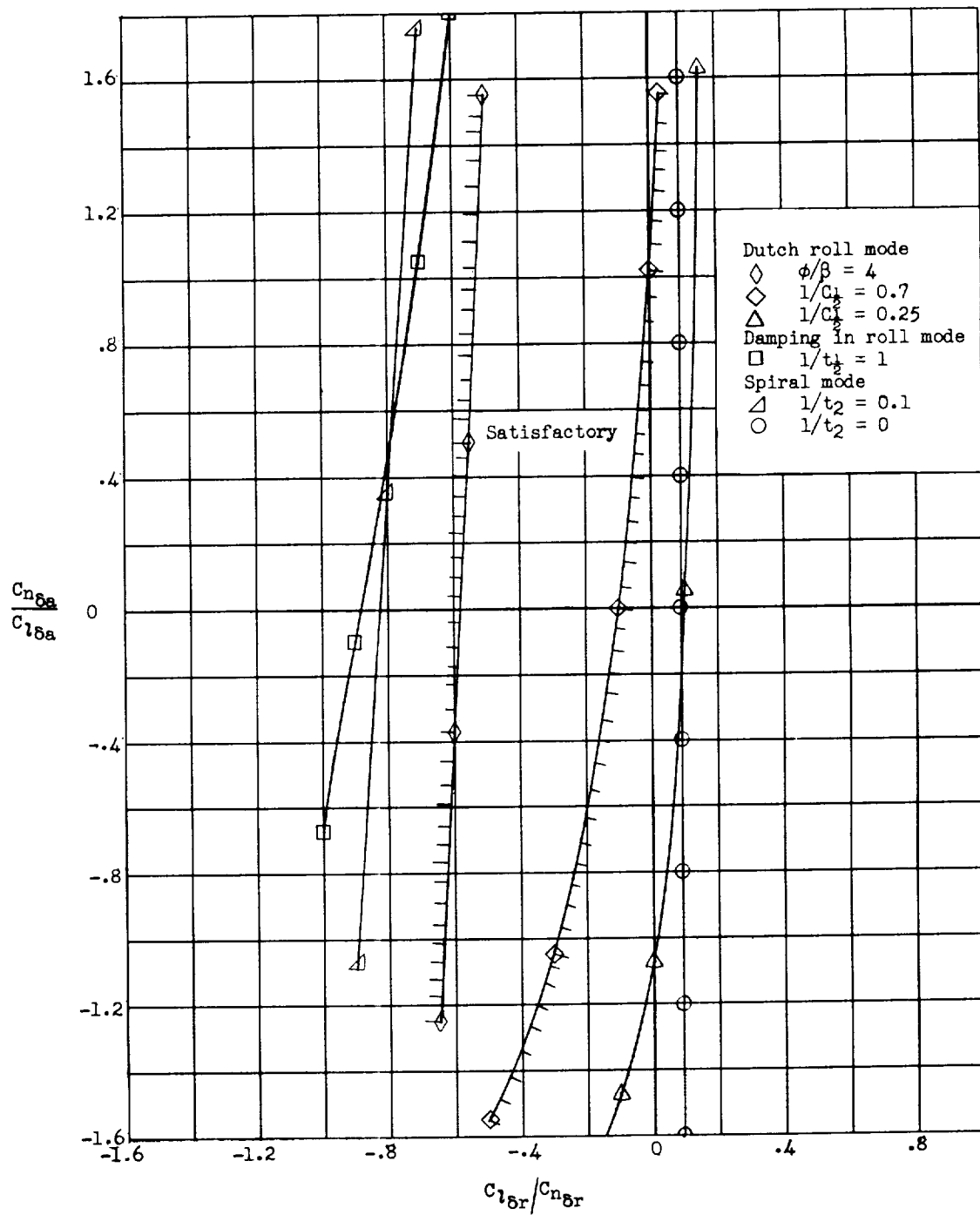
(d) Yawing-velocity response to a 10° rudder step function.

Figure 3.- Concluded.



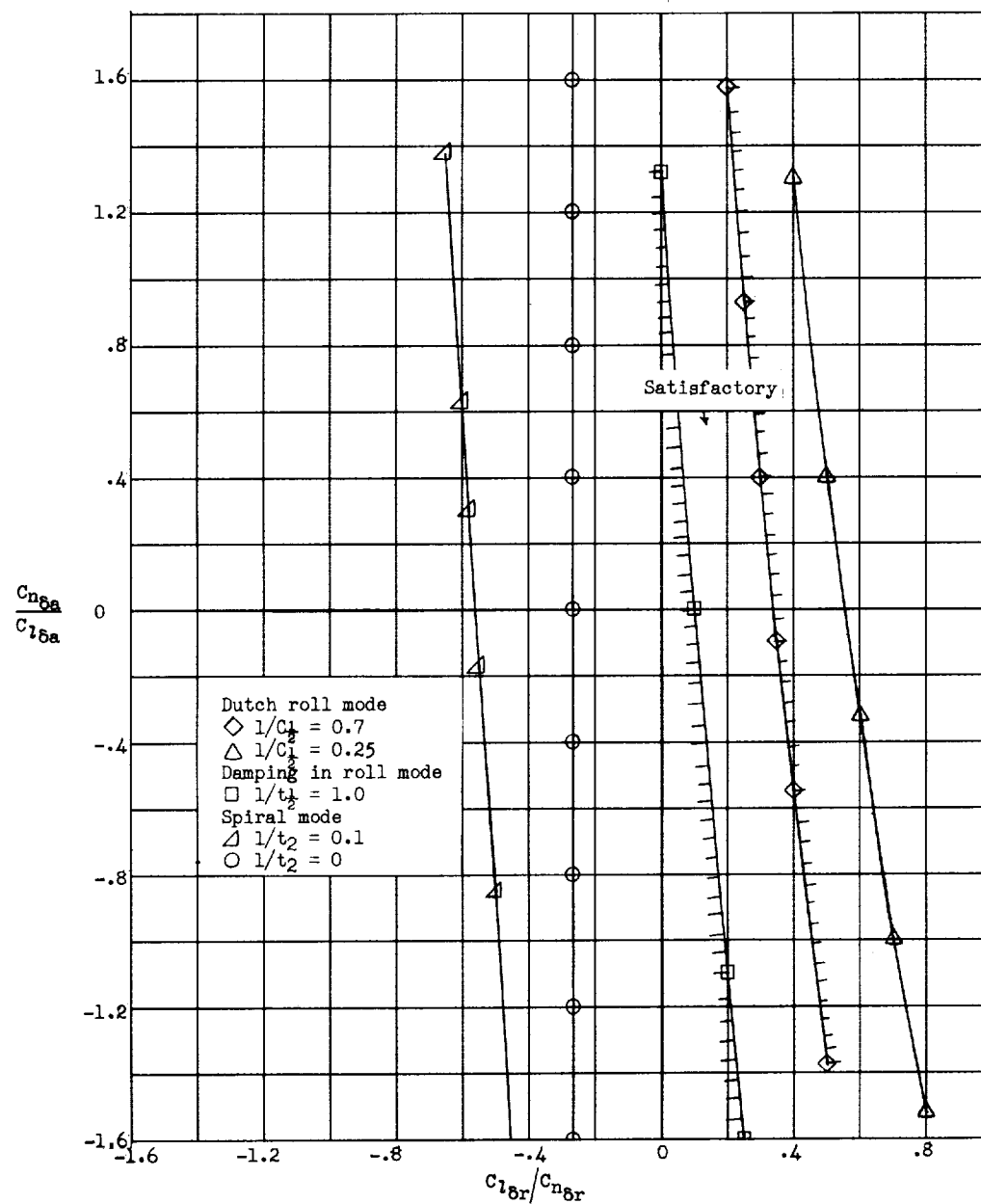
(a) Roll-damper gain of $k_1 = 0.05$.

Figure 4.- Effect of cross-control derivatives on the lateral characteristics of a high-speed airplane with negative effective dihedral and automatic dampers. $k_2 = 0.366$; $M = 6$; $h = 125,000$ feet; $C_{l\beta} = 0.027$; $C_{n\beta} = 0.31$.



(b) Roll-damper gain of $k_1 = 0.2$.

Figure 4.- Concluded.



(a) Roll-damper gain of $k_1 = 0.05$.

Figure 5.- Effect of cross-control derivatives on the lateral characteristics of a high-speed airplane with a decreased directional derivative and automatic dampers. $k_2 = 0.366$; $M = 6$; $h = 125,000$ feet; $C_{l\beta} = -0.027$; $C_{n\beta} = 0.1$.

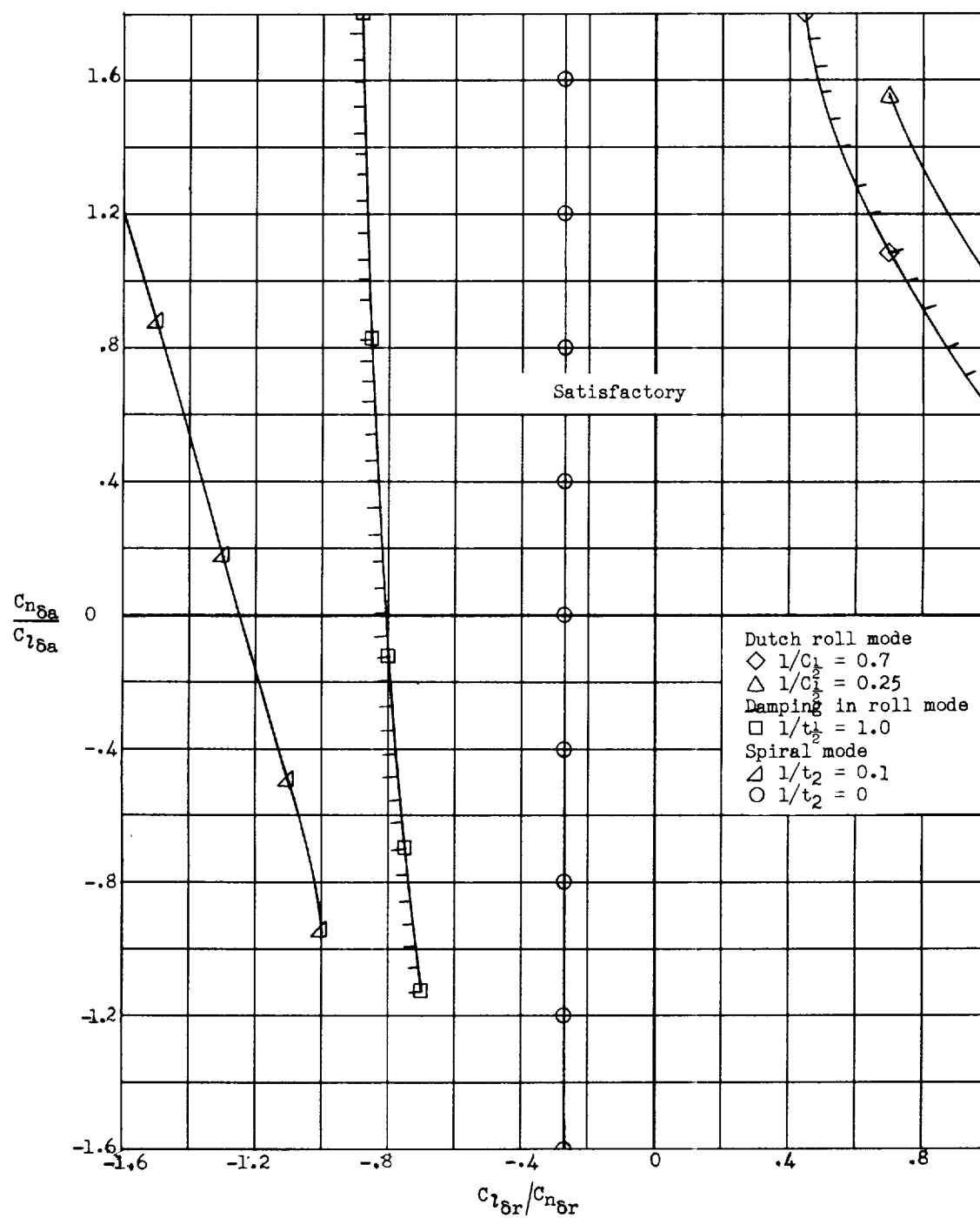
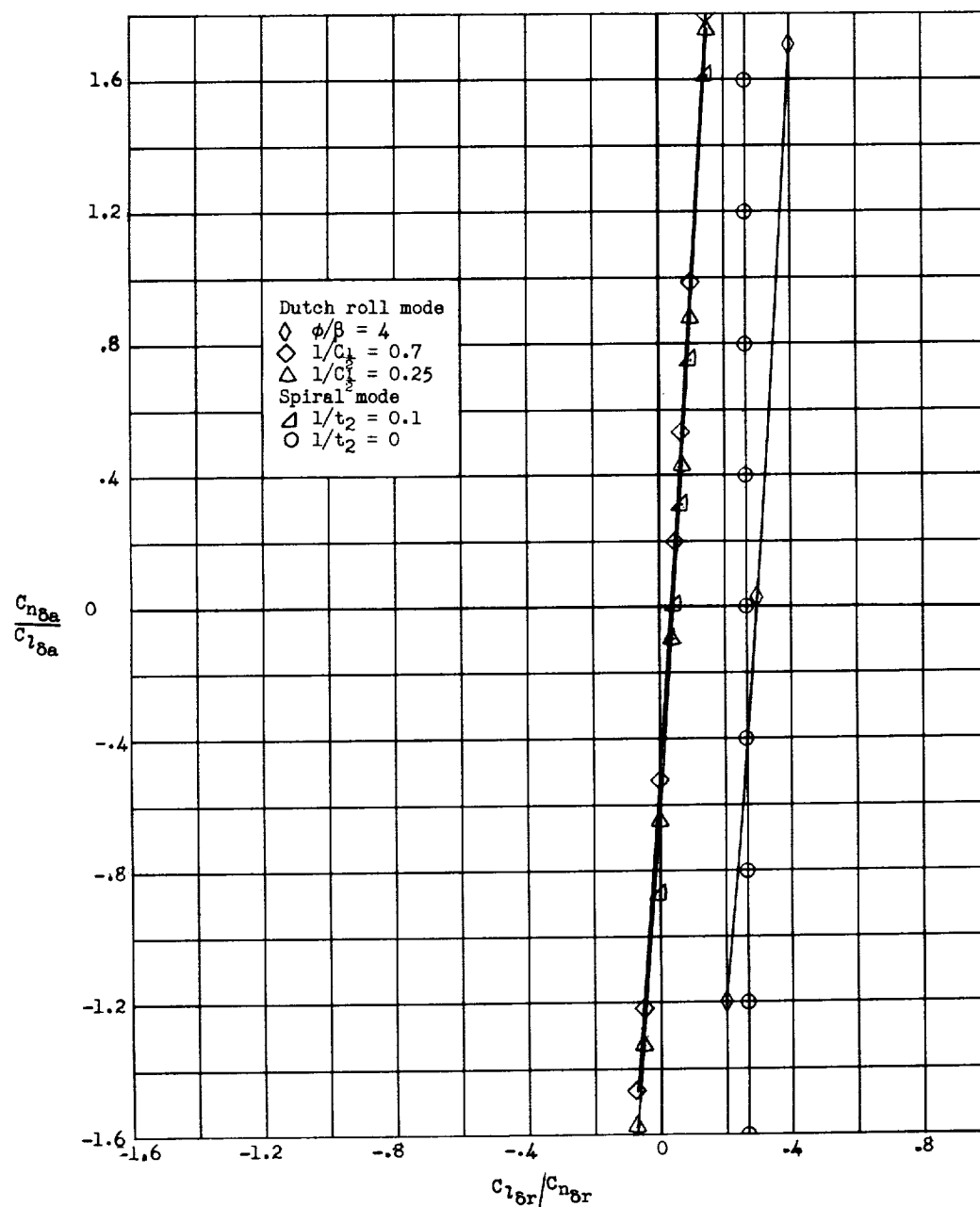
(b) Roll-damper gain of $k_1 = 0.2$.

Figure 5.- Concluded.



(a) Roll-damper gain of $k_1 = 0.05$.

Figure 6.- Effect of cross-control derivatives on the lateral characteristics of a high-speed airplane with a decreased directional derivative, a negative effective dihedral, and automatic dampers.
 $k_2 = 0.366$; $M = 6$; $h = 125,000$ feet; $C_{l\beta} = 0.027$; $C_{n\beta} = 0.1$.

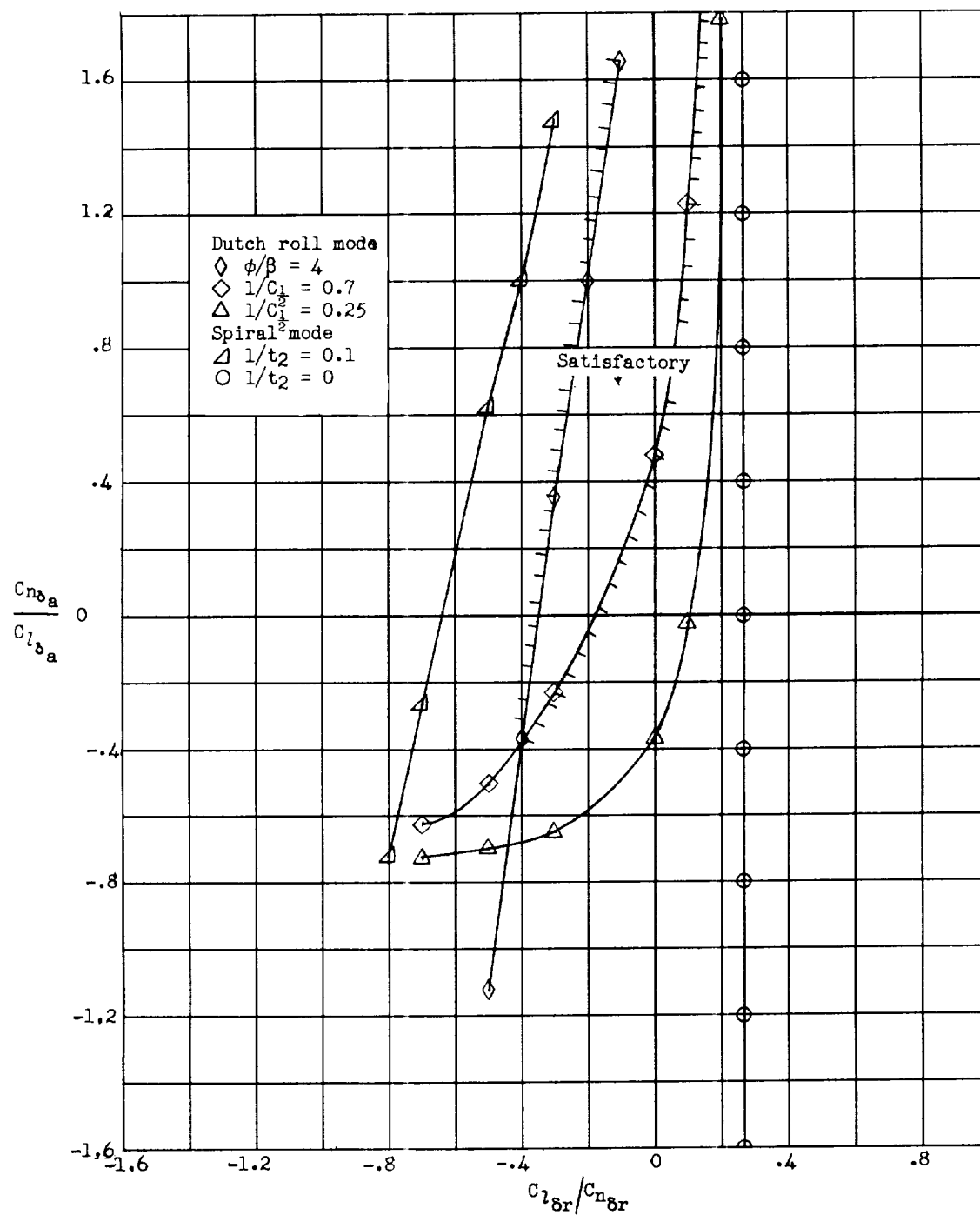
(b) Roll-damper gain of $k_1 = 0.2$.

Figure 6.- Concluded.

